A simple approach to measuring moment of inertia of body segments and fixturing used in biomechanical experiments

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ABSTRACT – The center of gravity (CG) and mass moment of inertia (MOI) of body segments and fixturing used in experimental post mortem human subject (PMHS) research can be measured quickly, easily, and without specialized equipment, using fundamental physics concepts. In this study, the accuracy of CG and MOI measurements made with simple equilibrium and physical pendulum experiments was evaluated. Using these methods, the measured CG and MOI of an anthropomorphic test device (ATD) head and neck that was mounted in fixturing used for PMHS head rotation testing were within 1% of the values measured by a high-precision gas bearing torsional pendulum system specifically designed for accurate CG and MOI measurements. The results suggest that these relatively simple measurement methods can be reasonable alternatives for accurate CG and MOI measurements in a biomechanics lab environment.

INTRODUCTION

Average mass properties of body segments have been compiled [1-3] but subject-specific measurements of properties such as segment center of gravity (CG) and mass moment of inertia (MOI) can provide valuable information in biomechanics research. These properties can be used to explain variation in dynamic response among test subjects or for accurate representation of test subjects in computational models. Subject-specific mass properties such as CG and MOI can also provide crucial information for dialing in input parameters to produce consistent kinematics among test subjects of varying sizes.

Measurement of component CG and MOI can be accomplished with specialized equipment like gas bearing torsion pendulums but those precision measurement systems require tightly-controlled operating conditions and may not be available during time-sensitive PMHS biomechanical testing.

Early biomechanics researchers used simpler techniques involving equilibrium balances, suspension techniques, and physical pendulum methods, as well as analytical geometry-based estimation [4]. The pendulum methods used in early MOI measurements by Dempster [5] and others are based on the concept that for small oscillations, the period (T) of a physical pendulum is dependent on MOI (*I*), as well as its mass (m) and the distance from its CG to the center of rotation (d), as shown in Equations 1 and 2 [6]. These equations account for the component of gravitational acceleration (g) in the direction of the pendulum's motion, which is proportional to the sine of the pendulum's angle from vertical (θ), assuming that the sine of that angle is equal to the angle. However, that assumption is reasonable only for small angles. As a result, Equations 1 and 2 apply to *small oscillations only.*

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$$T = 2\pi\sqrt{\frac{I}{mgd}} \quad \text{(Eqn 1)} \qquad I = mgd\left(\frac{T}{2\pi}\right)^2 \quad \text{(Eqn 2)}$$

Experimentally, precise measurement of the period is more difficult for small oscillations than for large oscillations. For large angle oscillations, the period (T) in this relationship can be corrected using the empirically-developed correction in Equation 3 for each pendulum oscillation [7]. Substituting $T_{corrected}$ from Equation 3 for T in Equation 2 allows MOI to be estimated with large-angle pendulum testing.

$$T_{corrected} = \frac{T}{1+\frac{\theta^2}{16}+\frac{11}{3072}\theta^4} \quad \text{(Eqn 3)}$$

Although physical pendulum techniques make it possible to measure biomechanical mass properties without specialized equipment, the accuracy of these experimental methods needs to be evaluated. This study reports on methods used to determine CG location and MOI using the equations above and data that can be collected in a PMHS lab environment. The accuracy of the measurements was evaluated by comparing repeated CG and MOI measurements made on an ATD head/neck mounted in a rotation fixture using these techniques to measurements made with a high-precision system, purpose-built for mass property measurement.

METHODS

The following procedures were developed to determine the CG and MOI of a PMHS head and its associated fixturing prior to, or immediately after, head rotation testing. For this evaluation, an ATD head and neck were used as ballast in place of post mortem tissue.

CG of the head/neck and cage

To identify the CG location of the head/neck and the fixturing ("cage") that grips the head during rotation tests, the head/neck and cage are suspended from a 9.5 mm diameter shaft that rotates freely in low-friction bearings. Pairs of clips fixed to the front and rear of the cage allow it to hang freely from the shaft in stable equilibrium (Figure 1) in two different positions.

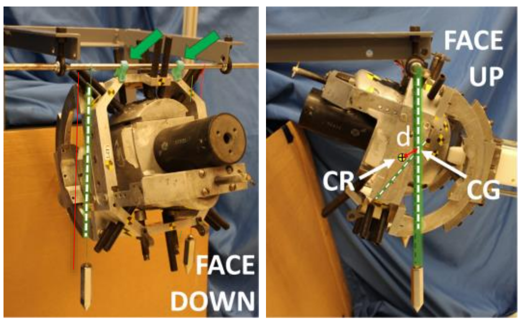

Figure 1. ATD head, neck and rotation cage hanging freely from clips at cage rear (left) and cage front (right)

In each position, a plumb bob is used to mark a vertical line directly down from the suspension point. The intersection of the vertical lines drawn from each suspension point corresponds to the combined CG of the head/neck and cage since the CG will always hang directly below the suspension point at equilibrium. The distance (d) from the CG location to the center of rotation (CR) to be used in pendulum testing is needed for Equation 2.

In case of asymmetry, the CG location is determined on both sides of the cage and the final CG location of the head/neck and cage system in the mid-sagittal plane of the head is estimated as an average of the locations measured on each side of the head.

MOI of the head/neck and cage

For pendulum testing of the head/neck and cage, the cage is fastened to the left end of a rotation shaft mounted in low-friction bearings allowing the head/neck and cage to rotate freely in the sagittal plane about the Y-axis, i.e., nodding (Figure 2). The rotation shaft location corresponds to the center of rotation used in PMHS head rotation testing.

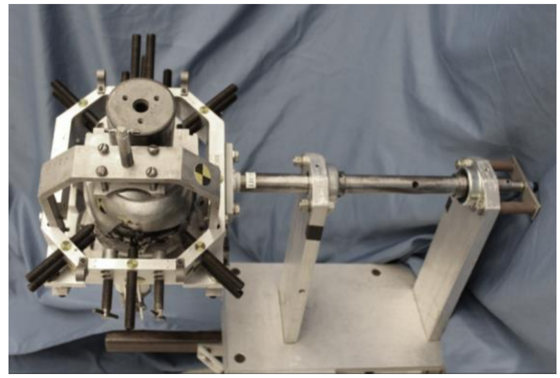

Figure 2. ATD head and neck in rotation cage (viewed from above) mounted to the left end of rotation shaft

The head/neck and cage system is rotated 90 degrees counterclockwise from its naturally hanging position and released to allow it to swing like a pendulum. The rotation time-history is collected using a Sfernice 50ESC102 rotary potentiometer (Vishay, Malvern, PA) mounted to the right end of the MOI fixture's rotation shaft. The angle (θ) from vertical is sampled at 20,000 Hz for 10 seconds and filtered with the SAE J211 CFC180 filter (Figure 3). Overlapping time periods (T) are measured using the time of maximum or minimum θ for each oscillation (peak-to-peak), and the time when the pendulum passes through vertical at θ of 0 degrees (base-to-base). Peak θ is averaged across each period with Equations 4 and 5.

$$\theta_{(base)} = \frac{|\theta_1| + |\theta_2|}{2} \qquad \text{(Eqn 4)}$$

$$\theta_{(peak)} = \frac{|\theta_1| + 2|\,\theta_2| + |\theta_3|}{4} \qquad \text{(Eqn 5)}$$

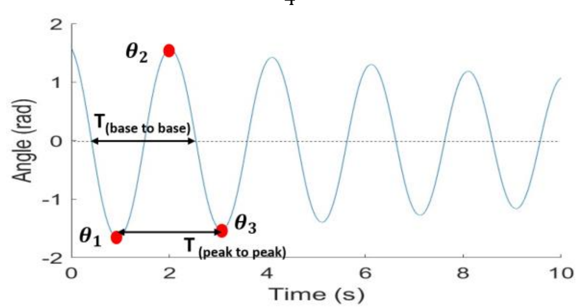

Figure 3. Measurement of time periods (T) and peak θ in configuration shown in Figure 2

Peak θ is used to apply the large angle correction (Equation 3) to each oscillation cycle. The resulting $T_{corrected}$ is averaged across all peak-to-peak and base-to-base oscillations and substituted into Equation 2 to estimate the combined MOI (I_Y) of the head/neck, cage, and the rotating parts of the MOI fixture shown in Figure 2. The previously-determined y-axis MOI of the rotating portions of the pendulum fixture (determined from geometry: MOI_F) and components of the cage (determined from isolated cage MOI testing: $\boldsymbol{MOI_C}$) can be subtracted from the measured MOI ($\boldsymbol{I_Y}$) calculated in Equation 2 to estimate the MOI of the head/neck and cage ($\boldsymbol{I_Y - MOI_F}$) and MOI of just the head/neck ($\boldsymbol{I_Y - MOI_F - MOI_C}$) about the rotation axis.

Accuracy Evaluation

The accuracy of the CG and MOI measurements made using equilibrium and physical pendulum techniques was evaluated with comparison measurements using a KSR330-60 gas bearing torsion pendulum system (Space Electronics LLC, CT USA). The head/neck and cage system was mounted to the rotary table of the KSR330-60 via an adapter plate that positioned it for rotation about the same axis of rotation used in the pendulum method proposed in this paper. With the head/neck and cage system mounted on the KSR330-60's rotary table, the system determines CG location as a function of the moment required to rotate the measured component, and determines MOI based on its response to torsional oscillation.

RESULTS

Results are summarized in Table 1. Results were the same to 4 significant digits in repeat testing.

Table 1. Comparison of mass properties measured using proposed methods and KSR330-60 torsion pendulum

	CG to CR (d)	Mass MOI (I_Y)
Gravity-based methods		
Head/neck, cage, & MOI fixture (n=3)	13.20 mm	0.1417 kg-m^2
Head/neck only	-	0.0338 kg-m^2
Head/neck and cage	15.13 mm	0.1414 kg-m^2
KSR330-60 Torsion Pendulum		
Head/neck and cage	15.28 mm	0.1420 kg-m^2
Difference between methods		
	-0.15 mm (0.9%)	-0.0006 kg-m^2 (0.5%)

DISCUSSION

In the current study, the MOI of an ATD head and neck in a head rotation fixture, measured with a gas bearing torsion pendulum, was $0.1420\ kg\text{–}m.^2$ Under ideal conditions, the reported accuracy of the torsion pendulum can be as low as 0.0254 mm for CG and ± 0.1% of value + 8.78E-6 kg-m^2 for MOI. The level of precision in the torsion pendulum measurements in this study could have been limited by the precision with which the head/neck and cage assembly could be aligned with the device's rotation axis. However, the system's accuracy was expected to exceed the precision possible with the proposed simple experimental methods. Therefore, the torsion pendulum measurements were used as a benchmark to assess the accuracy and precision of the proposed methods. The CG and MOI measured using these relatively simple methods in a biomechanics lab environment were within 1% of the measurements made by the precision torsion pendulum and also repeatable (n=3), suggesting they are a reliable alternative for mass property measurement of PMHS body segments.

It has been proposed that alternative MOI calculation methods that rely on angular velocity or angular acceleration in addition to angle time-histories in physical pendulum tests have the potential to estimate MOI even more accurately [8]. However, these methods require the use of additional instrumentation, which increases the risk of altering the pendulum's behavior. Experiments by the current study's authors compared MOI results calculated using a rotary potentiometer and Equation 3 versus by equations that rely on additional instrumentation and found that the difference in results was negligible.

In the physical pendulum testing reported in this study, the head and cage were mounted to the MOI fixture such that the CR in MOI testing corresponded to the CR in corresponding head rotation testing. As a result, the calculated MOI is directly relevant to the rotation response of the head/neck and cage in rotation testing. The parallel axis theorem can be used to estimate the corresponding MOI about the CG of the tested segments, or any other point.

The approach evaluated in this study assessed MOI_Y for rotation in a single plane. If needed for other studies, MOI_X and MOI_Z would need to be evaluated separately. When applied to the measurement of post mortem body segments, the potential for non-rigid behavior needs to be considered. For example, care can be taken to minimize deformation during CG and MOI data collection or to stabilize segments with fixturing that can be accounted for in the final calculations [5]. These considerations also apply to mass measurements made with specialized equipment.

CONCLUSION

For measurement of the mass properties of body segments and test fixtures in biomechanical research, relatively simple gravity-based methods that can be performed in a PMHS test environment can be reliable alternatives to gold standard measurements using specialized equipment.

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